MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

ECT1026 – FIELD THEORY

(ALL)

15 OCTOBER 2016
9.00 a.m - 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 6 pages with 4 Questions only.

2. Attempt **ALL (4)** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

(a) Given vectors $\vec{A} = 2\hat{x} + \hat{y} - 3\hat{z}$, $\vec{B} = 3\hat{x} - 4\hat{y}$ and $\vec{C} = 2\hat{y} - 4\hat{z}$. Determine

(i) Unit vector, $\hat{a}$ [2 marks]
(ii) $\vec{A} \times \vec{C}$ [3 marks]
(iii) $\hat{y} \times \vec{B}$ [2 marks]

(b) Sketch the spatial variation of the following vector fields using arrows in which the length of the arrow is proportional to the strength of the field and the direction of the arrow is the same as that of the field's.

(i) $\vec{E} = 2x\hat{y}$ [2 marks]
(ii) $\vec{B} = 4r\hat{\phi}$ [2 marks]

(c) Sketch the outlines of each of the surfaces and find the area of each of the following surfaces using appropriate differential surface area, $d\vec{s}$.

(i) $r = 3;\ 0 \leq \phi \leq \pi/3; -3 \leq z \leq 3$ [4 marks]
(ii) $R = 3; 0 \leq \theta \leq \pi/3\ ; 0 \leq \phi \leq \pi$ [4 marks]

(d)(i) Give the expression of divergence theorem for a vector field, $\vec{B}$. [2 marks]

(ii) Given vector field, $\vec{B} = 8e^{-r}\hat{r} - 3z\hat{z}$. Find $\iiint \vec{\nabla} \cdot \vec{B} dv$ for the cylindrical region enclosed by $r = 3$, $z = 0$ and $z = 3$. [4 marks]

Continued .....

Question 2

(a)(i) State the Coulomb's law formulated in year 1785 and gives its mathematical expression. [2 + 2 marks]

(ii) Give the two stipulations of Coulomb's law based on the experimental evidence. [2 marks]

(iii) Find the electric field intensity at $A(-0.2, 0, -2.3)$ due to a point charge of +5 nC at $B(0.2, 0.1, -2.5)$ in air. All dimensions are in meters. [5 marks]

(b) Charge q_1 is uniformly distributed over a thin spherical shell of radius R_1 and charge q_2 is uniformly distributed over a second spherical shell of radius R_2 where $R_2 > R_1$. Sketch the suitable Gaussian surface and determine the electric field, $\vec{E}$ in the region $R_1 < R < R_2$ using Gauss's law. [2 + 4 marks]

(c)(i) Explain the physical significance of electric potential, V. [2 marks]

(ii) Describe the Kirchhoff's voltage law using differential electric potential. [2 marks]

(d) Find an expression for the capacitance C of a parallel-plate capacitor comprised of two parallel plates each of surface area S and separated by a distance l. The capacitor is filled with a dielectric material with permittivity of $\varepsilon_r\varepsilon_o$, where ε_r is relative permittivity and ε_o is permittivity of free space. [4 marks]

Continued .....

Question 3

(a)(i) State Biot-Savart Law. [2 marks]

(ii) With the aid of a diagram, apply a suitable hand-rule to determine the direction of the magnetic field produced by a straight wire carrying current in the positive y-direction. [2 marks]

(iii) State the reason why the magnetic flux density of an infinitely long straight current-carrying conductor has no component in the direction of the current? [1 mark]

(b) Using Ampere's Circuital Law, find the expression of the magnetic field intensity H inside a long, straight conductor of radius 5 mm carrying a uniform current density J of 250 A/m^2. [6 marks]

(c) A toroid is wound on a circular core of mean radius r with cross-sectional area of A. If there are N turns and the relative permeability of the core is μ_r, show that the inductance of the toroid is

$$L = \frac{\mu_0 \mu_r N^2 A}{2\pi r}$$ [10 marks]

(d) An air core toroid has a cross-sectional area of 10 mm^2. It has 1000 turns and the mean radius is 20 mm. Find its inductance. [4 marks]

Continued .....

Question 4

(a) Briefly describe the types of magnetism below and give an example of material for each of the type.

(i) Diamagnetism [2 marks]
(ii) Paramagnetism [2 marks]
(iii) Ferromagnetism [2 marks]

(b) Briefly describe how a non-magnetized ferromagnetic material would respond when it is exposed to an external magnetic field. [4 marks]

(c) The magnetic circuit in Figure Q4 has a cast steel core with dimensions as shown. It is required to establish a flux of 1.2 mWb in the air-gap of the central limb. The relative permeability of the core material is assumed infinity, $\mu_r = \infty$, and fringing effect is negligible.

(i) Draw an equivalent magnetic circuit. [4 marks]
(ii) Calculate the reluctance of the air gaps 1, 2 and 3. [6 marks]
(iii) Determine the required mmf of the exciting coil. [5 marks]

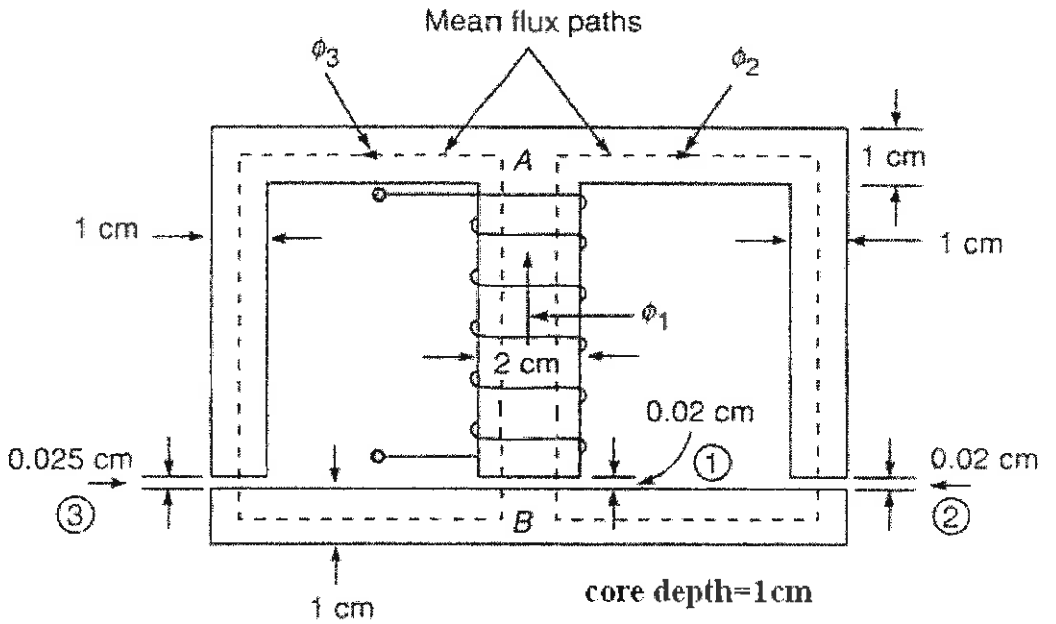

Figure Q4

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Physical Constants, Vector and Coordinate Transformation Relations

Elementary charge	e	1.60×10^{-19} C
Permittivity of free space	ε_0	8.85×10^{-12} F/m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x}dydz$ $d\vec{s}_y = \hat{y}dxdz$ $d\vec{s}_z = \hat{z}dxdy$	$d\vec{s}_r = \hat{r}rd\phi dz$ $d\vec{s}_\phi = \hat{\phi}drdz$ $d\vec{s}_z = \hat{z}rdrd\phi$	$d\vec{s}_R = \hat{R}R^2\sin\theta d\theta d\phi$ $d\vec{s}_\theta = \hat{\theta}R\sin\theta dRd\phi$ $d\vec{s}_\phi = \hat{\phi}RdRd\theta$
Differential volume	$dxdydz$	$rdrd\phi dz$	$R^2\sin\theta dRd\theta d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to Spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to Cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$